Errata for
"An Introduction To Partial Differential Equations"

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Who can understand his errors?
cleanse thou me from secret faults.
(Psalms 19,12)

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1. Page vii Line −8: replace indenpendent with independent.

2. Page 1 (1.1) should be

$$F(x_1, x_2, ..., x_n, u, u_{x_1}, u_{x_2}, ..., u_{x_1x_1}, ...) = 0,$$

3. In (1.10) and (1.14) replace: $\partial/\partial t$ with $\mathrm{d}/\mathrm{d}t$.

4. In (1.14) replace: $\mathrm{d}s$ with: $\mathrm{d}S$.

5. Page 10 (1.21) and lines 10 13: replace p_0 and ρ_0 with p^0 ρ^0.

6. Page 10 Line 18: replace $\vec{u}^1$, ρ^1, with ρ^1, $\vec{u}^1$.

7. In (1.30) it should be: $\frac{(\delta x)^2}{4\delta t} = k$.

8. Page 13 Line −5: replace $u \in C^4$ with $u \in C^2$.

9. (1.42) should be:

$$\mathrm{i}\hbar\frac{\partial u}{\partial t} = -\frac{\hbar^2}{2m}\Delta u + Vu,$$

10. In (1.48) replace D with Ω.

11. Page 26 Line 22: it should be "the German physicist Iris J. Runge (1888–1966)."

    Remark: Iris Runge was the daughter of the famous mathematician Carle Runge. Although she was at Göttingen University, she spent one year in the university of Munich. During this period she worked with Sommerfeld on asymptotic solutions of the wave equation. In their famous 1911 paper, Runge and Sommerfeld devised an asymptotic technique for wave propagation that became very useful in quantum mechanics and other fields. The method was later reinvented and improved by a number of people; it is called now the WKB method after three of these people.

12. Page 26 Line −4: please add: We assume that Γ is regular, that is $\Gamma \in C^1(I)$, and $(x_0(s)')^2 + (y_0(s)')^2 > 0$ for all $s \in I$.

13. The third line of (2.13) should be:

$$\frac{\mathrm{d}u}{\mathrm{d}t}(t) = c_0(x(t), y(t))u + c_1(x(t), y(t))$$

14. Page 32 Line −5 should be: Notice that the obtained solution is defined in

$$D = \{(x, y) \,|\, x < y\}.$$

    On the other hand, for the initial condition

$$u = \sin x, \quad \text{on} \quad y = x + x^2, \quad x \in \mathbb{R},$$

    the obtained solution for $s > 0$ can be extended only to the domain

$$D_1 = \{(x, y) \,|\, 0 < x < y\} \cup \{(x, y) \,|\, x \leq 0 \text{ and } x + x^2 + \varepsilon(x) < y\},$$

    where $\varepsilon(x) > 0$ for $x \leq 0$. Note also that the solution u is not differentiable at $(0, 0)$.

15. Page 35 Line −1: replace the x axis with $y = 1$.

16. Page 36 Line 15: replace a point s of Γ with $\Gamma(s)$.

17. Page 36 Line −11: It should be: Assume further that the transversality condition holds at a point s_0 on the initial curve.

18. Page 40 Line 20: replace $(0, 0, u)$ with u.

19. *Example 2.17.* Since the nonlinear system (2.85) for our problem is not uniquely solvable, it is possible to choose $y_t(0, s) = u_y = q_0(s) = -n/\sqrt{1 + s^2}$ instead of $y_t(0, s) = u_y = q_0(s) = n/\sqrt{1 + s^2}$. This different initial condition for the strip equations leads to another solution of the strip equations which satisfies the same Cauchy data of the problem, and hence it yields to another solution to the problem. It can be checked that this second solution represents a spherical wave starting from the point $(0, 2)$.

20. (2.99) should be

$$(x, y, u, p, q) = \left(s + \frac{4n_0}{\sqrt{5}}t, 2s - \frac{2n_0}{\sqrt{5}}t, 1 + 2n_0^2 t, \frac{2n_0}{\sqrt{5}}, -\frac{n_0}{\sqrt{5}}\right),$$

21. Page 42 Line 16: it should be "eliminate the t variable".

22. Page 42 Line −3: to avoid confusion, it is better to write: $u_x = (1 - yu_x)h'$.

23. Page 45 Figure 2.6: Replace t with y.

24. Page 46 Eq. (2.57) should be

$$u(x, y) = \begin{cases} 0 & x < \frac{y}{2}, \\ 1 & x > \frac{y}{2}. \end{cases}$$

25. Page 61 line 5, 8, and 11 should be

$$yu_x + uu_y = x.$$

26. Page 68 line −8 should be

$$u_{yy} = \frac{9}{4}(v_{qq} + 2v_{qr} + v_{rr}).$$

27. In theorems 3.5, 3.9, and 3.12 it should be written: There exists a local coordinate system.

    Remark: We prove only local existence of a canonical coordinate system.

28. Page 68 Line −6: it should be

$$u_{xx} + xu_{yy} = -9\left(\frac{q-r}{2}\right)^{2/3}\left[v_{qr} - \frac{v_q - v_r}{6(q-r)}\right] = 0.$$

29. Page 70 Line −13: it should be:

    $$x^2(y^2 v_{\eta\eta} + 2yv_{\xi\eta} + v_{\xi\xi}) - 2xy(v_\eta + xyv_{\eta\eta} + xv_{\xi\eta}) + x^2y^2v_{\eta\eta} + xyv_\eta + xv_\xi + xyv_\eta = 0.$$

30. Page 74 Line 9: it should be

$$u_{xx} - (1 + y^2)^2 u_{yy} - 2y(1 + y^2)u_y = 0.$$

31. Page 76 Line −12: it should be

    where $c \in \mathbb{R}$ ($c \neq 0$)

32. Page 92 Line 9: replace w_{xx} with $c^2 w_{xx}$.

33. Page 95 Line 19: replace $C^2([0,\infty)$ with $C^2([0,\infty))$.
34. Page 101 Line -17: replace initial boundary problem with initial value problem.
35. Page 101 Line -15: replace exists a solution with exists a nontrivial solution.
36. Page 109 Line 17: replace "clamped but free ends" with "two ends which are free to move vertically".
37. Page 122 Line -5: Should be 1892 (he was knighted in 1866).
38. Page 123 Line -13: should be:

$$E'(t) \leq -\int_0^\infty D(w_x)^2 \,\mathrm{d}x - 2\beta E(t) \leq -2\beta E(t).$$

39. Page 123 Line -10: should be:

$$E(T) \leq E(t)\mathrm{e}^{-2\beta(T-t)} \leq M\mathrm{e}^{-2\beta T}\mathrm{e}^{2\beta t}.$$

40. Page 132 Line 10: replace $-L$ with $-\frac{1}{r}L$.
41. Page 138 Line -9: should be: is called.
42. Page 139 Line 1: the word exists is redundant.
43. Page 138 Line -5: replace propositions with statements.
44. Page 142 Line -7: should be: Leibniz's product rule
45. Page 145 Line 11: should add the following remark:

    Remark: Lagrange identity and Proposition 6.20 hold true also for complex valued functions. More precisely, these results are valid when one considers the complex inner product space of piecewise continuous complex valued function on $[a,b]$ with respect to the inner product

$$\langle u,v\rangle_r := \int_a^b u(x)\overline{v(x)}r(x)\,\mathrm{d}x.$$

    We continue to denote this inner product space by $E_r(a,b)$. Recall that a complex inner product space satisfies the conjugate symmetry property $\langle u,v\rangle = \overline{\langle v,u\rangle}$ instead of the symmetry property $\langle u,v\rangle = \langle v,u\rangle$.

46. Page 152 Line -5: should be:

$$V = \{u \in C^2([a,b]) \mid u(a) = u(b),\ u'(a) = u'(b),\ u \neq 0\}.$$

47. Page 156 Line -10: should be: $-h(u(1))^2 \leq 0$.
48. In (6.91) it should be $\frac{1}{2m\pi}$ and not $\frac{1}{m\pi}$.

49. Pages 168 and 169, the phrase "with respect the related inner product" should be "with respect to the related inner product".

50. Page 169 Ex. 6.8: replace eigenfunctions with eigenvalues.

51. Page 170. Replace lines 14–15 with:

$$\begin{aligned} u_x(0,t) = u_x(\pi,t) = 0 \qquad & t \geq 0, \\ u(x,0) = 1, \quad u_t(x,0) = \cos^2\frac{\pi x}{2} \qquad & 0 \leq x \leq 2. \end{aligned}$$

52. Page 176. Replace lines 8 with: here Γ is any smooth closed curve satisfying $\Gamma = \partial D_1$, where D_1 is a domain that is fully contained in D.

53. Page 183 Line 7: it should be smooth bounded domain.

54. Page 185 Line 1: it should be $v_t - k\Delta v < 0$.

55. Page 185 Line 2: Replace 7.14 with 7.16.

56. Page 187 Line -5: it should be $\overline{D}$.

57. Page 193 lines -14 and -13: it should be $\sinh[\pi(1-y)]$

58. Page 197 Line -3: replace uniformly with uniformly in $\overline{B_a}$.

59. Page 199 Line -13: replace "in the theory" with "and reentrant corners in some problems".

60. Page 200 Line -1: replace $-2\tilde{f}$ with $-\tilde{f}$

61. Page 201, Eq. (7.77) should be:

$$\tilde{f}_n(r) = \begin{cases} r^3 - r & n = 1, \\ 0 & n \neq 1, \end{cases}$$

and the solution of Example 7.30 should be:

$$w(r,\theta) = \frac{1}{2} + (r^3 - r)\cos\theta + \frac{r^2}{2}\cos 2\theta.$$

62. Equation (7.85): it should be K_N.

63. Page 216 Line -5: replace "will not be given here" with "will not be given here (*hint:* show that $K \neq 0$ and use the strong maximum principle)".

64. (8.33) should be $\Delta N(x,y;\xi,\eta) = -\delta(x-\xi, y-\eta)$.

65. Page 223 Line 8: replace compact support with support.

66. Page 223 Line 9: replace ΔK with K_{xx}.

67. Equations (9.4) and (9.5): replace (twice in each equation) ∂ with d.

68. Equation (9.8) should be

$$\begin{aligned}
\frac{\mathrm{d}x_i}{\mathrm{d}t} &= \frac{\partial F}{\partial p_i} && i = 1, 2, \ldots, n, \\
\frac{\mathrm{d}u}{\mathrm{d}t} &= \sum_{i=1}^{n} p_i \frac{\partial F}{\partial p_i}\,, \\
\frac{\mathrm{d}p_i}{\mathrm{d}t} &= -\frac{\partial F}{\partial x_i} - p_i \frac{\partial F}{\partial u} && i = 1, 2, \ldots, n\,,
\end{aligned}$$

69. Page 235 Line 17: it should be 4.6.

70. Page 237 Line −4: replace u_{xx} with Δu.

71.

72. Page 240 Line −13: replace (twice) f with $\tilde{f}$.

73. Page 241 Line 16: it should be $(\xi_1 - x_1)^2 + (\xi_2 - x_2)^2$.

74. Page 244 Line 8: replace Multiply with Let u be an eigenfunction of (9.45)–(9.46) with an eigenvalue λ. Multiply.

75. Page 247 Line 15: it should be Diophantine equations.

76. Page 251 Line 8: it should be

$$\int_0^a r J_n^2(\sqrt{\lambda_{n,m}}\, r)\,\mathrm{d}r = \frac{a^2}{2}[J_n'(\sqrt{\lambda_{n,m}}\, a)]^2.$$

77. Page 252 Line 1: Replace coefficients are with coefficients are for $n \geq 1$.

78. (9.83) and (9.84): Replace $J_{n+1}(\alpha_{n,m})$ with $J_{n+1}^2(\alpha_{n,m})$.

79. Page 257 Line 15: replace exactly 2 with exactly 2 (except the principal eigenvalue which is always simple).

80. in (9.117): replace $T(0)$ with $T_m(0)$.

81. in (9.124): replace $T(0)$ and $T'(0)$ with $T_m(0)$ and ${T_m}'(0)$

82. Equation (9.147): one should replace $\hbar$ with $\hbar^2$.

83. Page 269 Line −11: replace "in space." with "in space and this bounded domain contains air which resists compression."

84. Page 271 Line 9: replace: ds with dσ.

85. Page 274 Line −5: replace it with its.
86. Page 287 Line −5, Page 303 Line −5, and Page 306 Lines $-3, -7, -12$: one should add min to the RHS.
87. Page 294 Line −8: replace coefficient, and $l(x,t)$ is the load on the string. with coefficient.
88. Page 295 Line −9: replace spring with string.
89. Page 298 Line 14: replace a continuous function with continuous functions.
90. Page 304 Line −8 should be: $\|u_n\|_{H_1(D)} < C^2$
91. Page 308 Line 7: replace in a given infinite-dimensional Hilbert with in a given Hilbert.
92. Page 309 Line −14: "is only obtained" with "is frequently obtained only".
93. Page 315 Line 7: should be $1 - 4\alpha \sin^2(L\Delta x/2) < 1$.
94. Page 315 Line −4: one should add: and the decay of the truncation error to 0 (as $\Delta x, \Delta t \to 0$) is faster than Δt.
95. Page 317 Line −14: it should be $A(n) = A(0)r^n$.
96. Page 317 Line −1: it would be better to write "we a priori need".
97. Equation (11.32): one should multiply the right hand side by 16.
98. Equation (11.75): replace $0 \leq x \leq 1$ with $0 < x < 1$.
99. Page 329 Line −5: replace domain with smooth bounded domain.
100. Page 336 Line 3: replace $(0, L)$ with $[0, L]$.
101. Page 330 Line 12: replace "mechanical engineers" with "civil engineers, and also by mechanical and aerospace engineers".
102. Page 339 Line −8: should be

$$u(x,y) = y\frac{1 - y^{y-x/y}}{x - y^2}.$$

103. Page 350 Line −7: should be $[0, \pi]$.
104. Page 351 Line 3: should be $\displaystyle\int_0^1 \frac{\mathrm{d}x}{1+x^2}$.

Stylistic Typos

1. In (1.31) omit the comma after 0.

2. In (1.33) omit the comma after $1/k$.

3. In (1.50) omit the comma after $\beta_2(t)$.

4. In (2.11) omit the comma after $u_0(s)$).

5. In (2.92) it would help the reader if we write explicitly

$$J|_{(0,s_0)} = \frac{\partial(x,y)}{\partial(s,t)}|_{(0,s_0)} = x_s(0,s_0)y_t(0,s_0) - y_s(0,s_0)x_t(0,s_0)$$
$$= x_0'(s_0)F_q(P_0) - y_0'(s_0)F_p(P_0) \neq 0,$$

6. In (2.100) omit the comma after 1.

7. Page 62, Line 13: omit the comma after 1.

8. Page 92 Line -1: replace: $\cos(t)$ with: $\cos t$.

9. Page 107 Line 8: it is better to write

$$\sum_{n=1}^{\infty} [(u_n)_t - (u_n)_{xx}] = 0,$$

10. (5.43) omit the comma before $n = 1$.

11. (5.87) should be:

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} - \beta V, \qquad \text{where } D := \frac{1}{C_s(r_i + r_e)}\ , \quad \beta := \frac{1}{r_s C_s}\ .$$

12. Page 138 Line -5: replace "propositions" with "statements".

13. Definition 6.14 is better formulated as follows:

   1. Let $v \in V$. The (formal) series

   $$\sum_{n=1}^{\infty} \langle v, v_n \rangle v_n$$

   is called the *generalized Fourier expansion* (or in short, Fourier expansion) of v with respect to the orthonormal system $\{v_n\}_{n=1}^{\infty}$.

   2. If $\lim_{k\to\infty} \|v - \sum_{n=1}^{k} \langle v, v_n \rangle v_n\| = 0$ exists, we write

   $$v = \sum_{n=1}^{\infty} \langle v, v_n \rangle v_n,$$

   and we say that the *Fourier expansion of v converges in norm* (or on average, or in the mean) to v.

14. Page 157 Line 7: omit the comma after $\sin \frac{(2n-1)\pi x}{2}$.

15. In (6.92) put more space between $F(x,t)$ and $a < x < b$.

16. Page 165 Line 11: " we check what are the PDE" should be " we check what the PDE are".

17. Page 165 Line -16: replace $((p(x)v_x)_x + q(x)v)$ with $[(p(x)v_x)_x + q(x)v]$.

18. Page 178 Line -12: replace $C^2(D) \bigcap C(\bar{D})$ with $C^2(D)$.

19. Page 178 Line -7: replace $M = \max$ with $M := \max$ and $L = \max$ with $L := \max$.

20. Page 186 Line 13: "sense even" should be "sense that even".

21. Page 195 Line 9: replace ds with $\mathrm{d}s$.

22. Page 195 Line 13: replace the first $\partial_2\Omega$. with $\partial_2\Omega$, .

23. Page 202 Line 1: replace "Consider $r < \tilde{a} < a$. Since the series converges uniformly there," with "Fix $r < a$ and $\theta \in [0, 2\pi]$. Since the series converges uniformly for $\varphi \in [0, 2\pi]$,"

24. (7.81) is better written as

$$w(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} K(r,\theta;a,\varphi)h(\varphi)\,\mathrm{d}\varphi \qquad 0 \le r < a,\ 0 \le \theta \le 2\pi,$$

25. Page 213 Line 13: add: (where $c =$ constant).

26. Page 213 Line -5: replace the Laplace operator $-\Delta$ with the operator $L := -\Delta$.

27. Page 234 Line -7: replace "Radially" with "Spherically".

28. Page 254 Line -6: replace P with $P(t)$.

29. In (9.110) replace $(P_n^m(\cos\phi))^2$ with $[P_n^m(\cos\phi)]^2$.

30. (9.116) should be

$$u(x,t) = \sum_{m=1}^{\infty} T_m(t)v_m(\vec{x}), \quad F(x,t) = \sum_{m=1}^{\infty} F_m(t)v_m(\vec{x}), \quad f(x) = \sum_{m=1}^{\infty} f_m v_m(\vec{x}).$$

31. Change the order of (9.126) and (9.127).

32. (9.143) Replace ∂B_a with $[0,\pi] \times [0, 2\pi]$.

33. In (9.148) omit the comma after $R = 0$.

34. In (10.37) Should be $J :=$.

35. Page 294 Line -9:

$$E_p = \int_a^b \left[d\left(\sqrt{1+u_x^2} - 1\right) - lu\sqrt{1+u_x^2} \right] \mathrm{d}x,$$

36. Chapter 11. The method is called Finite Element Method and not Finite Elements Method.

37. In (11.27) omit the comma after $f(x_i, y_j)$.

38. In (11.29) omit the comma after $= 0$.

39. In (11.79) add a comma after 1.